

Trading Up – Two for One

NOTE: Children should always be given ample time to experiment, notice, and wonder before they are provided an explanation.

Always engage children with our two favorite questions:

What do you notice?

What do you wonder?

Resist the urge to answer any questions children have while exploring. Instead, respond back with questions to children and let them make sense of the world. Sample questions you might use: What do you think? Do you notice any patterns? What could we change? Can we test something else? What can we try next? If children ask a testable question, which they could answer by doing an experiment, talk through with them how they might design a test to help answer their question. As much as possible and within reason, let them actually test their questions by trying the experiments they propose.



Learning Objectives

Children will...

- practice counting.
- use conversions to trade up Unifix cubes for those with higher values.
- subitize Unifix cubes by the conversion factor.

Vocabulary (See **What the heck? Explanation of Science** at the end for definitions.)

convert

exchange

more or less

Materials

Five different colors of
Unifix cubes

Foam dice

Trading Up Game
Boards and Player
Boards (laminated if
possible)

Dry erase markers

Optional: Trading Up –
Two for One – Binary
Number player cards

You can also just draw the game board on a small handheld whiteboard. Other colored objects like building blocks or bingo chips can be used in place of Unifix cubes.

Key Question

How many of the final colored (orange, in the example) pieces can you collect?

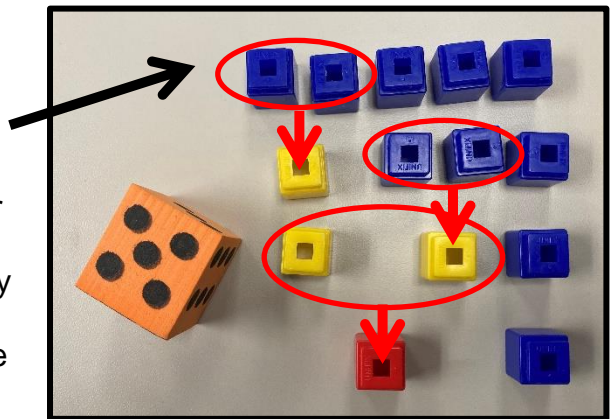
Advanced Teacher Preparation

1. Print out and laminate the Trading Up – Two for One game board.
2. Set up the game board with two Unifix cubes of the first color (blue) on the first line that equal one Unifix cube of the second color (yellow).
3. On the second line, put two Unifix cubes of the second color (yellow) that equal one Unifix cube of the third color (red).
4. On the third line, put two Unifix cubes of the third color (red) that equal one Unifix cube of the fourth color (green).
5. On the fourth line, put two Unifix cubes of the fourth color that equal one Unifix cube of the fifth color (orange). See example on the first page and note that the colors do not have to match the ones shown exactly.

Notice and Wonder Developmentally Appropriate Practice

Two to four children can play at a time for each game board.

1. The first child rolls the die and collects that number of Unifix cubes in the first color on the game board. (Example shown to the right: if they rolled a five they would get five blue cubes.)
2. Children should collect their blue cubes after they roll and then convert to other colors if they can (Example: With five blue cubes they can turn in two to get a yellow, and turn in another two get a yellow. Because they have two yellows, they would then convert those to a red cube. So five blue cubes convert or trade up to one red cube and one blue cube.)
3. Once the child is done with all of their possible conversions, their turn is over. If they have converted all their cubes, they should never have more than one of any color. If they have two of a color, they can trade up those two for one of the next color.
4. Children take turns rolling the die and trading the cubes up.
5. After each child has had a turn, one round has been completed. You can decide how many rounds children should play (5 rounds is a good start) or you can decide how many of the final pieces are required to win the game. (Example: Whoever gets two orange cubes first wins the game.)
6. As children are playing, you can prompt them as necessary to help them identify conversions they can still make with their cubes.
7. At first children will need more support, but once they learn the game, they will be able to set it up and play on their own.





Children should notice...

- how many cubes they need to convert to new colors.
- they can skip picking up the first color because they already know two blues will convert to a yellow.
- as they acquire the final cube color and stack them, they can compare who has more or less by comparing the length of their Unifix cube towers.

Differentiating Developmentally Appropriate Practice

Younger children will need more direct, concrete practice with one-to-one correspondence to make the conversions. They will not be able to make the jump from four cubes to two cubes of the next color. Instead, they will need to keep counting two of the first color trades up to one of the second color and so on. Let them keep playing in a very concrete way.

Older children can make the leaps to converting multiple cubes at time.

Extensions for Additional Learning

As always, ask the children throughout the lesson what they notice and what they wonder. If their wonder questions are testable, as much as possible and within reason, let them actually test their questions by trying new setups or rules or versions of the games.

See below for examples of what they might wonder and experiments they might do to test their wonderings.

- I wonder if we can add five lines of conversions?
- I wonder if the conversions always have to be the same number of cubes or could the first line require two, then next three, the next four, and so on?
 - Let them try it! Once they learn the game, any way they want to change it as long as the new rules are clear, should be encouraged.

References

Very special thanks to Áleph in Lima, Peru where a 4-year old mathematician taught the original Two for One game to Carmen while she was serving as an ITEN Fellow on a trip supported by the Organization of American States.

SAFETY CONCERNS

Don't let children put the cubes or die or bingo chips in their mouth. As with anything small, there is always a choking hazard.

AZ Early Learning Standards

Math Standard- Strand 1: Counting & Cardinality Concept 4: Counts to Tell

Number of Objects: The child uses number words and counting to identify quantity

Math Standard- Strand 2: Operations & Algebraic Thinking Concept 1: Explores

Addition and Subtraction: The child recognizes addition as adding to and subtraction as taking away from.

Trading Up: Conversions Start Here

What the heck? **Explanation of the Math** (Vocabulary in bold.)

Games like this are the foundation for **converting** units of measurement later and understanding number systems. **Exchanging** cubes based on the rules set up on the game board will help children practice counting and subitizing. It also helps students with one-to-one correspondence and making comparisons of quantities – having either **more or less** than what someone else has.

The original Two for One game is actually modeling how the binary number system works. This can be made more apparent by engaging students to use the Trading Up – Two for One – Binary Numbers player cards to organize their cubes a little differently. After students have played using the Binary Numbers player cards, they can move on to trading up 10 for 1. These extended lessons are included on the following pages.

To better understand positional notation and binary numbers, this video does a great job explaining both in a quick, accurate way: <https://youtu.be/LpuPe81bc2w>

Trading Up Binary Numbers – Roll to Win Game

It turns out the two for one conversion used in this game is actually modeling the binary number (base 2) system where we use zeros and ones to represent numbers. This is like a physical version of Exploding Dots 1 \leftarrow 2 Machine found here:

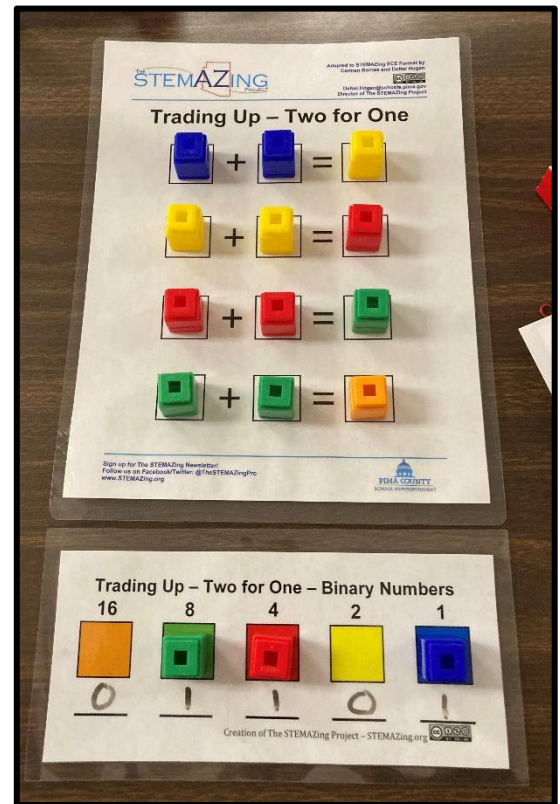
<https://www.explodingdots.org/station/l1S1>

Advanced Teacher Preparation

1. Print out Trading Up – Two for One – Binary Numbers player boards, cut into three player boards, and laminate. You can have 2-4 players so you may need to make another copy of the player boards.
 - If you are using the same color pattern in the example, you can print the color-coded player boards.
 - In the case you may not have a color printer or the same colored Unifix cubes to match the colors on the game board, print out the blank player cards, color them in with the corresponding colors, and then laminate them.
2. Set up the game board as shown to the right (the same as in the first game).

NOTE: For older students, the game board with the conversions shown is not actually needed.

The colors on the player board and knowing you are trading up two for one from right to left is enough.

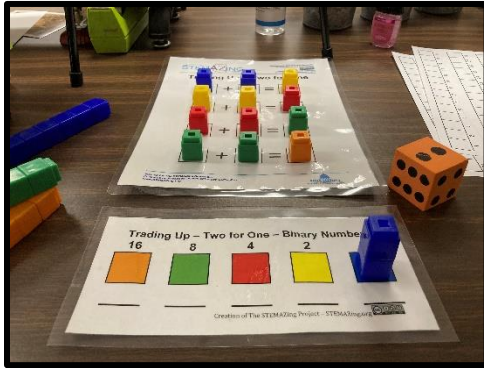


Notice and Wonder Developmentally Appropriate Practice

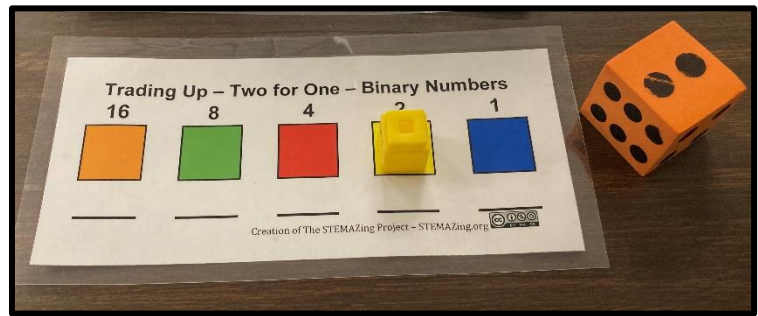
The rules are exactly the same as the Two for One game. Students roll the die, collect that number of blue cubes, and then trade up two for one. The only difference is that they will now keep their cubes on their player boards and do a bit more noticing and wondering about the numbers! On the following pages, you will find an example of a player board for one child playing this game. This child first rolls a 2, then a 6, and then a 5. The end of this game can be to earn two orange cubes, which would require trading up to the next position, 32, which is not included on this version of the player cards.

Example for a player rolling in three different rounds.

Roll One - 2

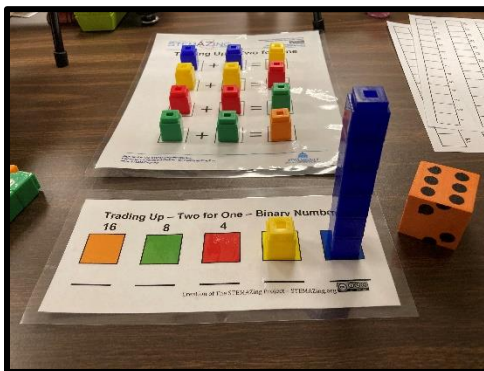


Child collects two blue cubes.

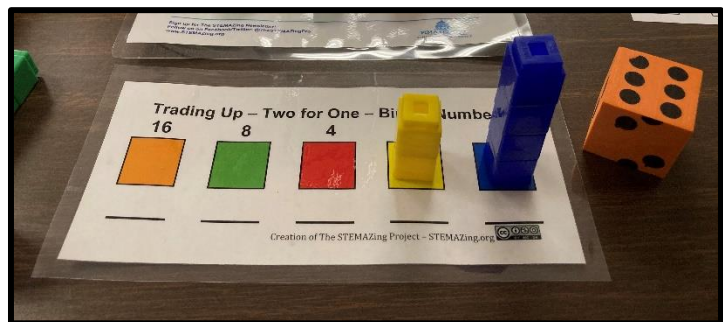


Child trades up two blue cubes for one yellow cube in the next position.

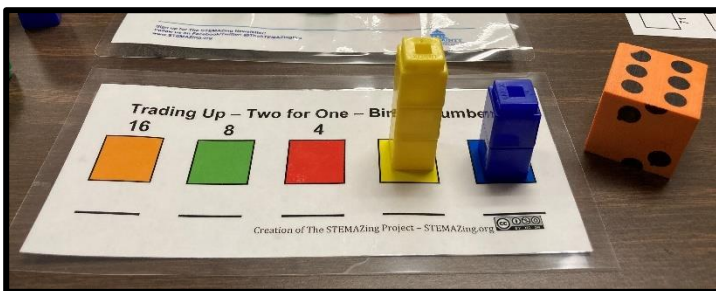
Roll Two - 6



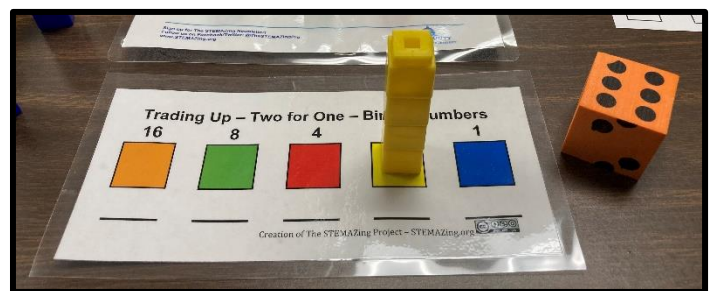
Child collects six blue cubes.



Child trades up two blue cubes for one yellow cube and stacks the new yellow on the existing one from the last round.

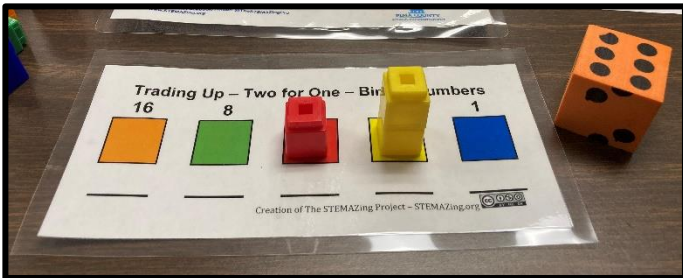


Child trades up another two blue cubes for a yellow cube.

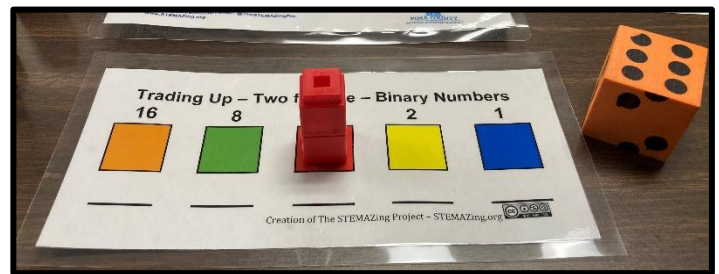


Child trades up final two blue cubes for another yellow cube.

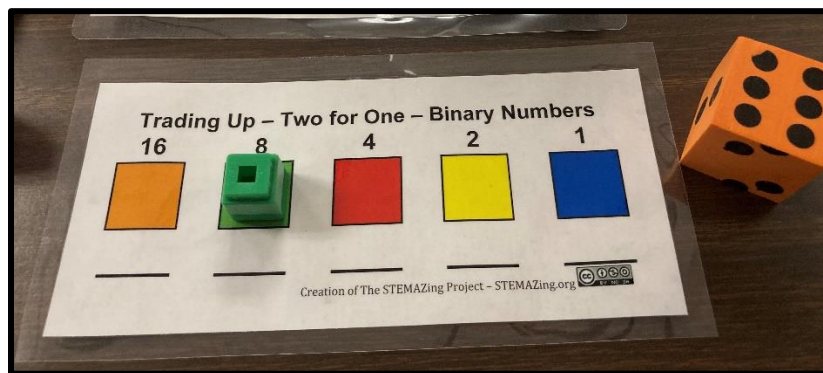
Continuing conversion...



Child converts two yellow cubes into one red cube.



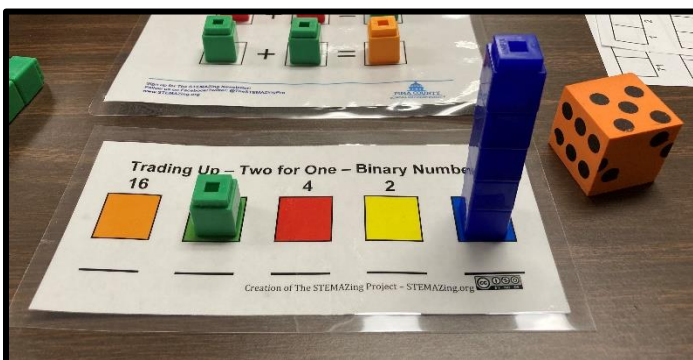
Child converts two yellow cubes into another red cube.



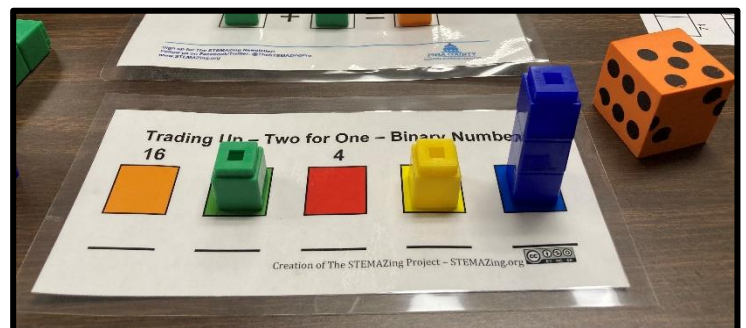
Child converts two red cubes into one green cube.

NOTE: From the child's first two rounds $2 + 6 = 8!$ If 8 is written in binary it would be 0 1 0 0 because there is only a cube in the fourth position – the eighths place.

Roll Three – 5

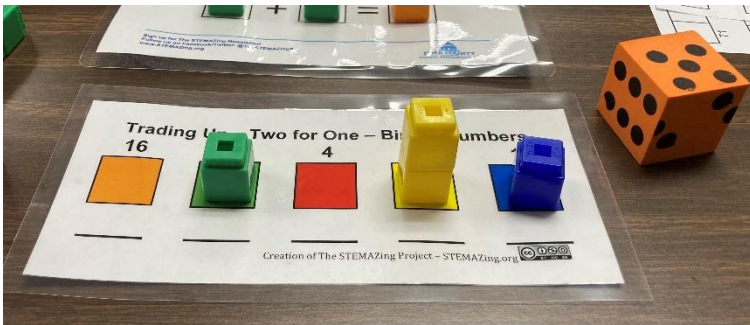


Child collects five blue cubes.

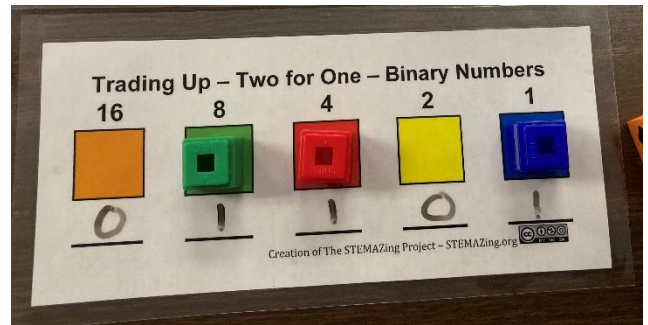


Child converts two blue cubes to one yellow cube.

Continuing conversion...



Child converts two blue cubes to another yellow cube.



Child converts two yellow cubes to a red cube.

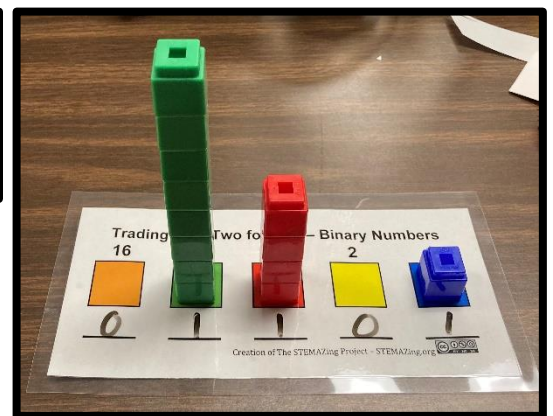
NOTE: From the child's first three rounds $2 + 6 + 5 = 13$. If 13 is written in binary it would be 0 1 1 0 1 because there are only cubes in the first, third, and fourth positions from the right. Note also that according to binary, counting the positions that are on with a 1, you can confirm that $1 + 4 + 8 = 13$.

Winning the Binary Number Game

The winner can either be the first child to get an orange cube or the first child to get two orange cubes or whatever metric you want!

The winner could also be the child with the biggest number at the end of 5 rounds. In that case, the children would likely have to work backward to figure out what their number is after five rounds.

If 1 child ended with the board to the right, the child could figure out their total by building a tower of 8 green cubes, 4 red cubes, and just the 1 blue cube. They could then combine them together into a tower and either count them or use the Unifix cube number line to figure out the value they have. They could also just compare the height of their towers to those of the other players. (NOTE: the cube towers don't have to match the colors of the positions – but it looks nice!)



Older students can just add the numbers together for the positions that have cubes on them. Again, $8 + 4 + 1 = 13$.





Trading Up Binary Numbers – Card Game

It turns out the two for one conversion used in this game is actually modeling the binary number (base 2) system where we use zeros and ones to represent numbers. This is like a physical version of Exploding Dots $1 \leftarrow 2$ Machine found here:

<https://www.explodingdots.org/station/l1S1>

Advanced Teacher Preparation

1. Print out Trading Up – Two for One – Binary Numbers player boards without color, cut into three player boards, and laminate. You can have 2-4 players so you may need to make another copy of the player boards.
2. Print out double sided and flipped on the short edge, the binary number cards (pages 18-25). Cut the cards apart so they are approximately the same size.
3. Give each child who is playing a player board.
4. Shuffle the binary cards and place them in the middle between all the players. The base 10 number should be facing up and not the binary number.
5. Ideally each player will need access to 35 Unifix cubes each. If you don't have that many, you can take out cards from the deck. For instance, if you only have 20 cubes per player, then you would only have cards 0-20 in the deck. The color of the cubes doesn't really matter for this game.

Notice and Wonder Developmentally Appropriate Practice

Playing Together

In this version of the game, children can work together to figure out the binary equivalent to the number showing on the top card. They can help each other if they get stuck and so on.

Playing in Secret

In this version of the game, children keep their work and answers hidden from the other players with a box or folders as barriers. Each child works independently on their conversion and then everyone shares their answers at the same time. If there are any mistakes, they can be talked through.

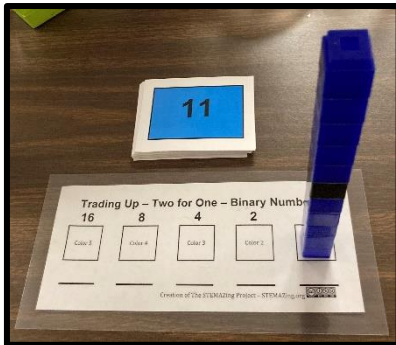
Playing for Speed

In this version of the game, children race to figure out the binary equivalent number to the number showing on top of the deck. As soon as they are done and have written the zeroes and ones on their player board, they can either slap the table or call out "Check it!" If they are the first to call out, they flip the card from the top of the stack over and compare it to their answer. If they match, that player gets to keep the card. If they don't match, then that card is put in the discard pile. You may need to find a way to hide the card deck from view so children don't start to work on the next number. Or you can let the others start which would give them an advantage for winning the next card.

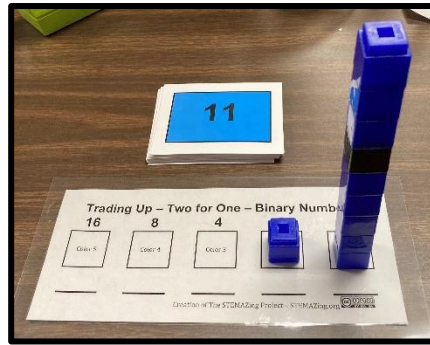
How Children Convert from the Number to its Binary Equivalent

Example 11

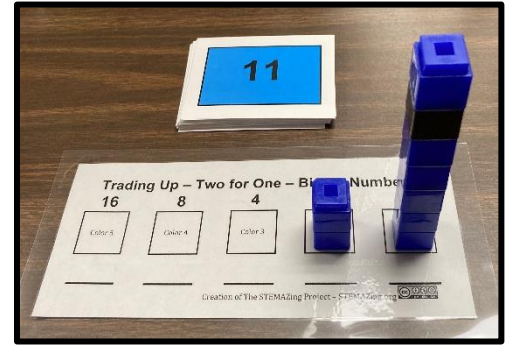
If 11 is the top card on the deck, then a child might do the following steps to convert it to binary.



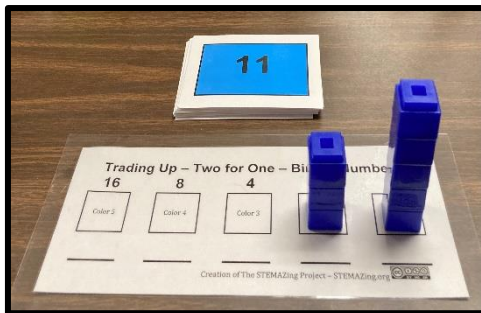
Make a tower of 11.



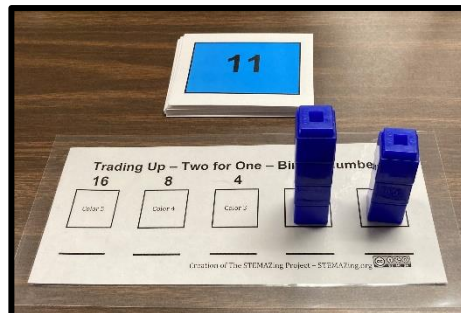
Trade two for one from position 1 to 2.



Trade two for one from position 1 to 2.



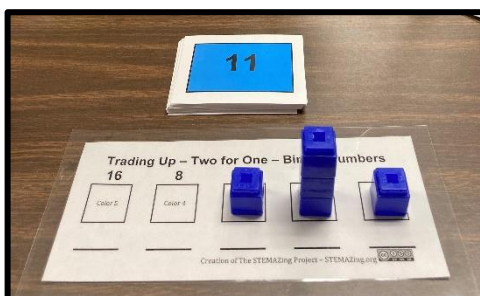
Trade two for one from position 1 to 2.



Trade two for one from position 1 to 2.



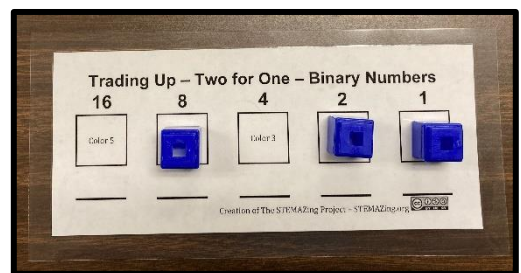
Trade two for one from position 1 to 2.



Trade two for one from position 2 to 4.



Trade two for one from position 2 to 4.



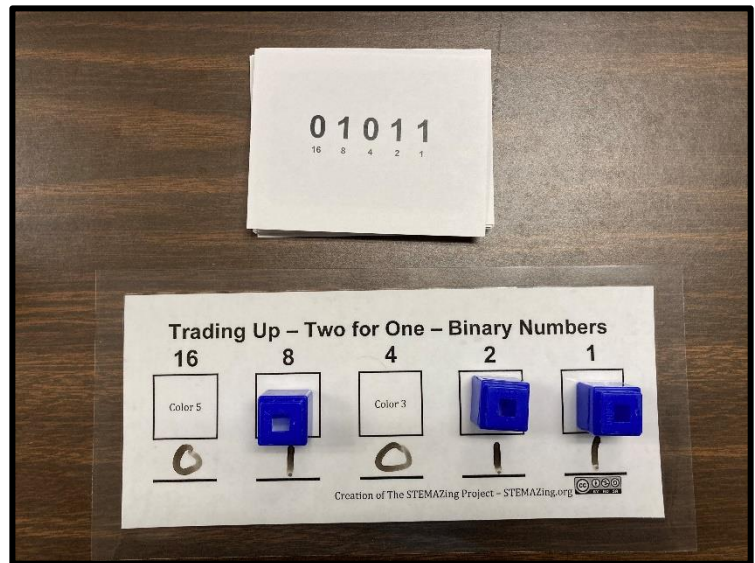
Trade two for one from position 4 to 8.

As soon as the child has figured out the binary number equivalent and written zeroes under positions without cubes and ones under position with cubes, they can call out “Check it!” and grab the top 11 card off the deck. They then verify if it matches their answer. If it does, they keep the card. If it doesn’t, they put the card in a discard pile.

The person with the most cards at the end wins the game!

For younger children, If you have enough Unifix cubes, you could have them use the color-coded player board.

You would need 35 blue cubes, 18 yellow cubes, 9 red cubes, 4 green cubes, and 2 orange cubes for each player. (NOTE: This is why switching to the blank boards and not worrying about the color is recommended for this game.)

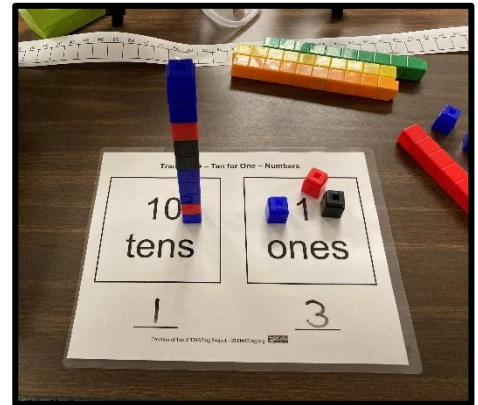


Trading Up – 10 for 1

After children have played two for one, they can move on to 10 for 1. This will allow them to work with our base 10 system and as long as children can count up to 10, they can use these activities to start to develop their understanding of positional notation.

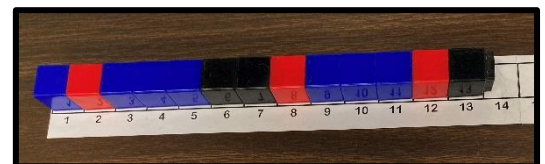
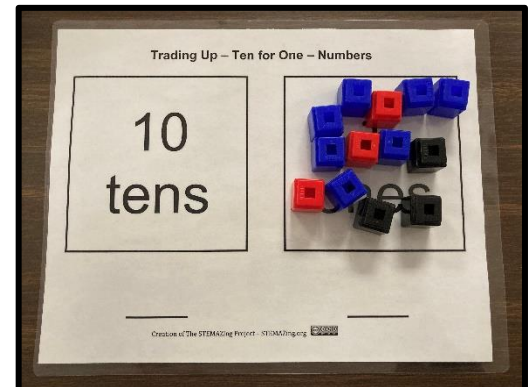
Advanced Teacher Preparation

1. Print out and laminate either the two position (ones and tens) or three position (ones, tens, and hundreds) Trading Up – 10 for 1 pages.
2. Print out (preferably on cardstock) and tape together the Unifix cube number line. This can also be taped up a wall or across a table if you want.



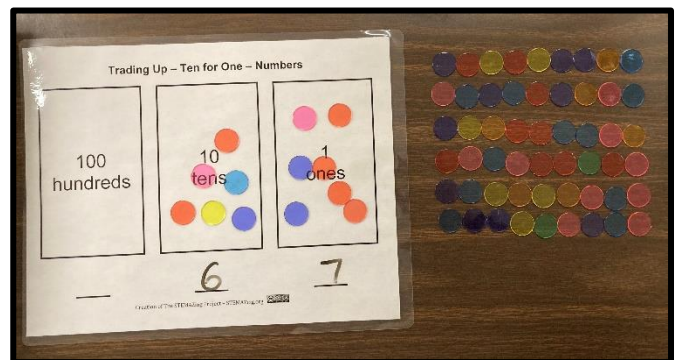
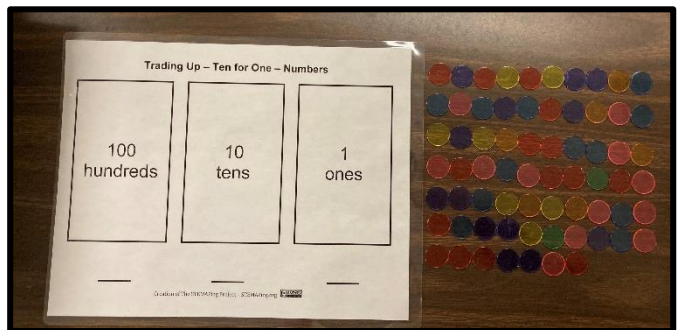
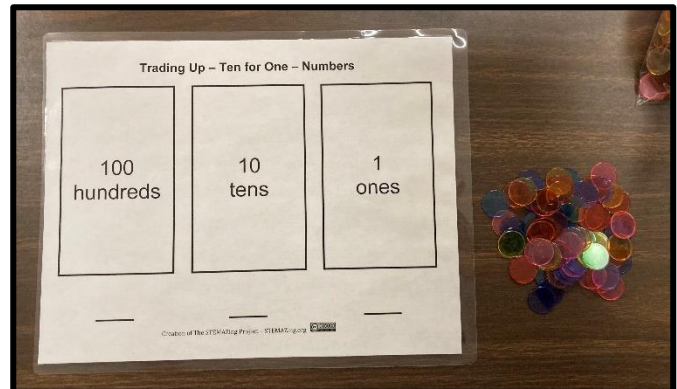
Notice and Wonder Developmentally Appropriate Practice

1. Children can be given some items to count. In this case, to the right it shows Unifix cubes but anything can be used from beans to bingo chips to bear counters or whatever you have on hand.
2. Teach children the new rule is 10 for 1 instead of 2 for 1. Unifix cubes from the ones place can be stacked to make towers of ten, as shown at the top of the page. If you are using counters which are not stackable like the Unifix cubes (beans or bingo chips) then once they have counted ten out of the ones position, they would simply take one of counters and put it in the 10 tens position to represent the 10 for 1 trade up. For younger children, you could make this more concrete by using one color of counter for the ones place and a different color counter for the tens place.
3. Once children have traded up all the tens they have, then they can write the number on the page using a dry erase marker. In this case, they had one tower of ten and three ones. So, the number of Unifix cubes was 13.
4. If you are using Unifix cubes, you can have children make a tower with all of the cubes and check it against the number line as shown.



10 for 1 Advanced

1. Older children can be engaged to count larger numbers of objects. In this case, they are given a pile of bingo chips.
2. They can arrange the chips into groups of ten and a group for what is left over.
3. Then, they can take one marker from each group of ten to put in the tens place and all the markers left over that didn't make a final group of ten and put them in the ones place.
4. Now, they can count the chips in each place and write the number using a dry erase marker below.

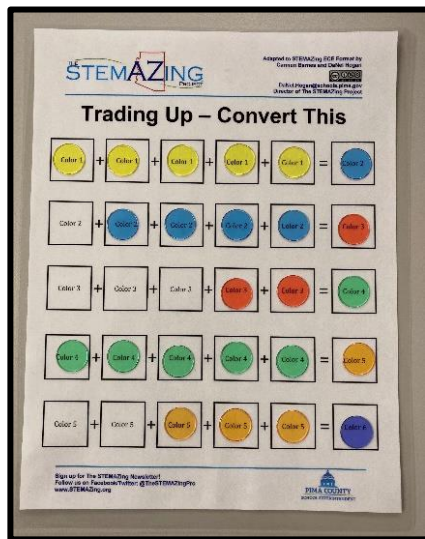


10 for 1 Game

1. Similar to the 2 for 1 Game students can use either the two position or three position 10 for 1 player boards. They can roll one or two dice each turn, take that many bingo chips (or other counters) and add them to their board in the ones place.
2. If they have more than 10 counters in the ones place, they can trade them up to the tens place and then eventually trade up 10 tens to one counter in the hundreds place!

Trading Up – Advanced

Children can use the more advanced conversion board to add more cubes (or bingo chips as shown to the right) to what is required to trade up.



Note that the conversions do not always have to be the same at each level as shown to the left – similar to 12 inches equal a foot and 3 feet equal a yard and so on.

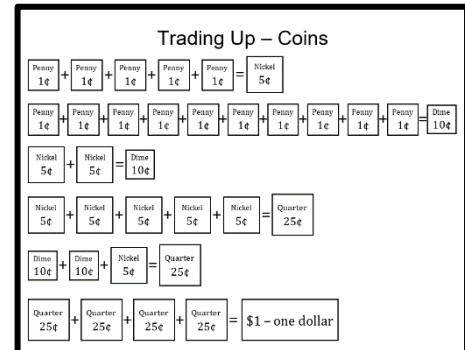


Trading Up – Coins

Children can learn about coin values and conversions using the Trading Up – Coins mat.

They can first put the coins in the correct places to show how many of one equal another. They can then, like in the other conversion games, use a single die or two dice to roll for pennies and then convert them up to other coins until they get all the way up to a dollar or two. You decide the rules!

(Special thanks to Irma Ibarra for this idea using coins!)



Trading Up – Two for One

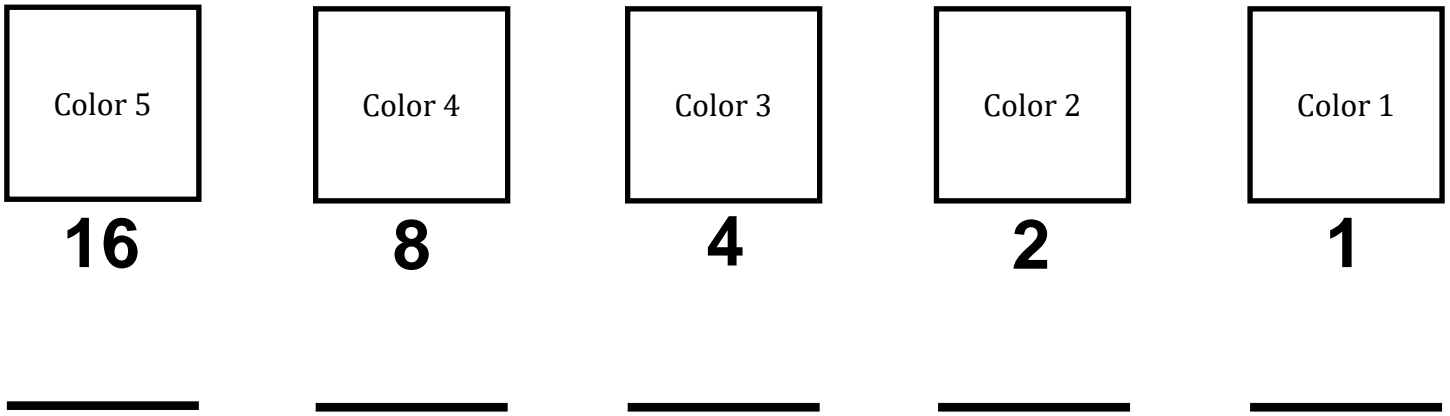
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$$\begin{array}{|c|} \hline \text{Color 4} \\ \hline \end{array} + \begin{array}{|c|} \hline \text{Color 4} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{Color 5} \\ \hline \end{array}$$

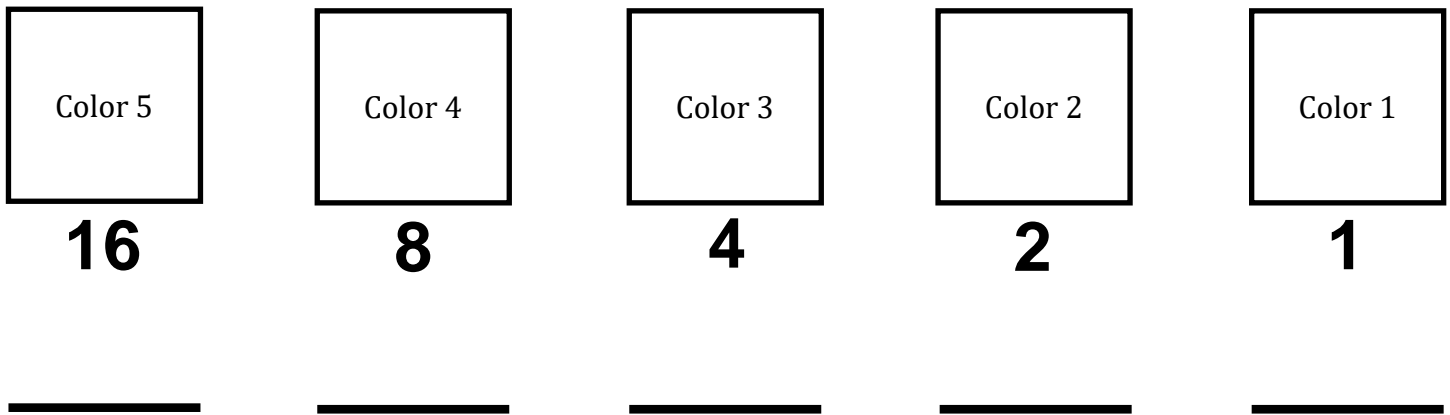
Trading Up – Two for One – Binary Numbers



Creation of The STEMAZing Project – STEMAZing.org



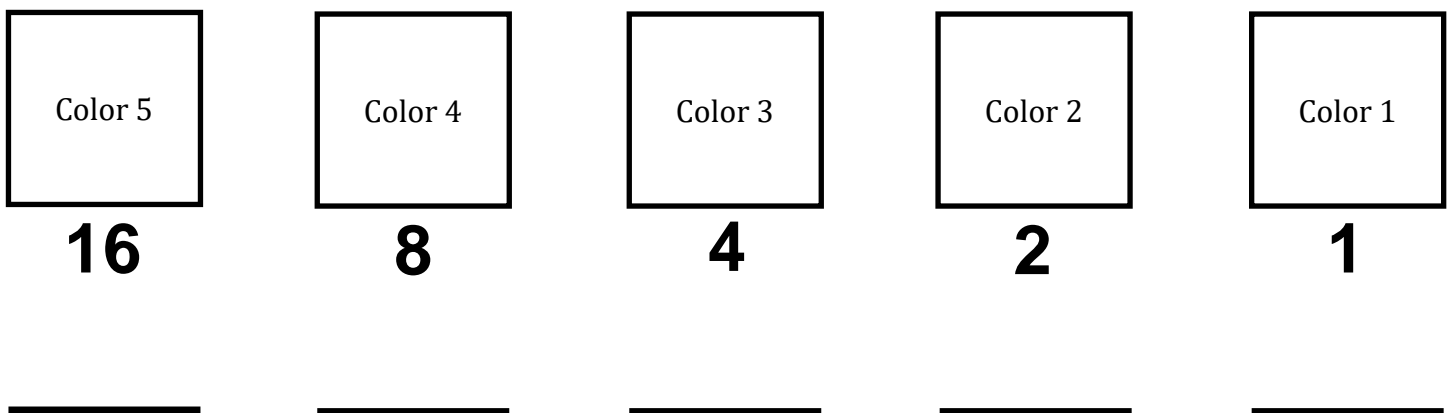
Trading Up – Two for One – Binary Numbers



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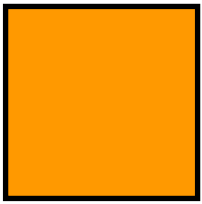
Trading Up – Two for One – Binary Numbers



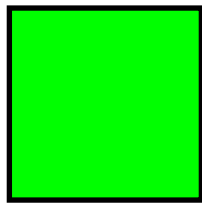
Creation of The STEMAZing Project – STEMAZing.org



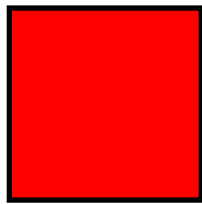
Trading Up – Two for One – Binary Numbers



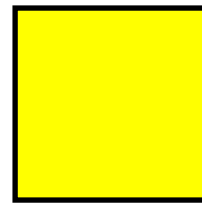
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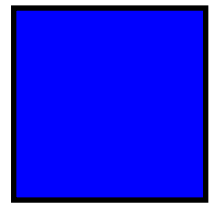
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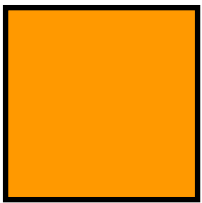
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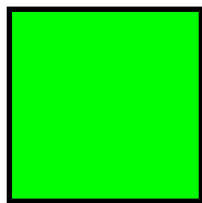
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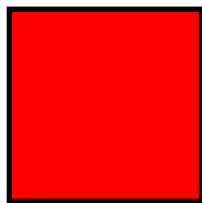
Trading Up – Two for One – Binary Numbers



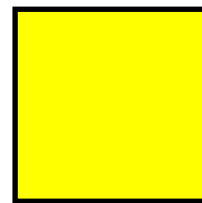
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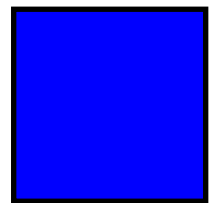
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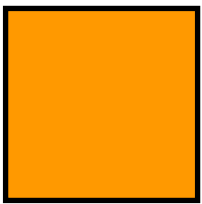
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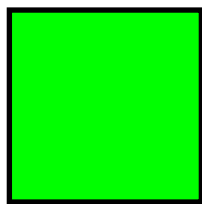
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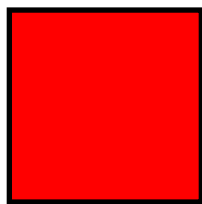
Trading Up – Two for One – Binary Numbers



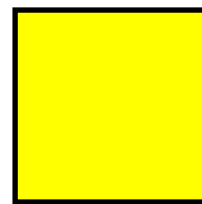
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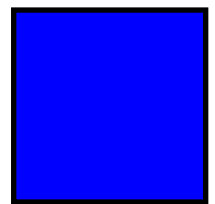
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4



2



1



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16 8 4 2 1

0 0 0 0 1
16 8 4 2 1

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16 8 4 2 1

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1 2 3 4 5 6 7 8 9 10 11 12 13 14

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15 16 17 18 19 20 21 22 23 24 25 26 27 28

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85 86 87 88 89 90 91 92 93 94 95 96 97 98

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99 100 101 102 103 104 105 106 107 108 109 110 111 112

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113 114 115 116 117 118 119 120 121 122 123 124 125 126

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127 128 129 130 131 132 133 134 135 136 137 138 139 140

Trading Up – 10 for 1

10
tens

1
ones

Trading Up – 10 for 1

100
hundreds

10
tens

1
ones

Trading Up Advanced

$$\boxed{\text{Color 1}} + \boxed{\text{Color 1}} + \boxed{\text{Color 1}} + \boxed{\text{Color 1}} + \boxed{\text{Color 1}} = \boxed{\text{Color 2}}$$

$$\boxed{\text{Color 2}} + \boxed{\text{Color 2}} + \boxed{\text{Color 2}} + \boxed{\text{Color 2}} + \boxed{\text{Color 2}} = \boxed{\text{Color 3}}$$

$$\boxed{\text{Color 3}} + \boxed{\text{Color 3}} + \boxed{\text{Color 3}} + \boxed{\text{Color 3}} + \boxed{\text{Color 3}} = \boxed{\text{Color 4}}$$

$$\boxed{\text{Color 4}} + \boxed{\text{Color 4}} + \boxed{\text{Color 4}} + \boxed{\text{Color 4}} + \boxed{\text{Color 4}} = \boxed{\text{Color 5}}$$

$$\boxed{\text{Color 5}} + \boxed{\text{Color 5}} + \boxed{\text{Color 5}} + \boxed{\text{Color 5}} + \boxed{\text{Color 5}} = \boxed{\text{Color 6}}$$

