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Binary #MicDropMath Multiply This

Find all related files to print and videos for this lesson here:
https://stemazing.org/binary-mic-drop-math-multiply-this/

Recommended Order of Binary #MicDropMath Lessons:

While all these lessons can stand on their own, in a perfect world with eons of time to engage students, the following would be the recommended sequence:

- Binary #MicDropMath Patterns
- Binary #MicDropMath Build Your Own Cards
- Binary #MicDropMath Multiply This
- Human Computers – Creative Message Communication

These can all be found here: https://stemazing.org/binary-mic-drop-math-lessons/

The following lesson engages students to explore exceptionally old algorithms used by Russian and Egyptian people to multiply two numbers together. It turns out these two algorithms are connected to the binary number system, which also serves to demonstrate that the binary number system was not invented to be the language of computers. It has been around and used since ancient times! This could also be used as an integration point if students are studying either Russian or Egyptian cultures. This lesson can also serve as an introduction to technical writing.

Russian [and Egyptian] Multiplication – NumberPhile video: https://bit.ly/RussianX

Materials

Paper and writing utensils for each student
Printables (optional because this can all be done on paper)
* any step with an * after it denotes that there is a printable for that activity

Engage

Show the NumberPhile video and just let the students watch it all the way through once. Have them watch it again and write down what they notice and what they wonder.

Explore

1. Next, have students write the steps for the Russian algorithm.* You can use this link to show just the Russian Algorithm section of the video, it may take a few times, so they can get all the steps recorded: https://bit.ly/JustRussianX

2. Have students use their own directions to try one practice problem.* If they find something that is unclear, they should correct it.

3. Next, have students swap the steps they have written with someone else. They should try to follow the steps they have been given to complete a second practice

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problem. They should provide constructive feedback to the algorithm writer if there are ways the steps could be improved.

4. Once they have revised their steps, they complete four more practice problems. You can decide if you want to require them to use bigger numbers based on their level.

5. Repeat Steps 1-4 but this time for the Egyptian Algorithm. You can use this link to show just the Egyptian Algorithm section of the video as many times as needed: https://bit.ly/EgyptianX

6. You can have students debrief the algorithm writing.
   - Was it easier writing the steps for the Egyptian Algorithm compared to the Russian Algorithm. Why?
   - Did they use any of the same steps they had developed in the first algorithm for the second?
   - What did they learn from the first experience that made the second easier or faster?

Explain

Because we don't think in terms of binary numbers, students will often have to convert from base 10 decimal numbers into base 2 binary numbers. There are two different algorithms that come from the Russian and Egyptian multiplication algorithms. It does not call them out during the video but it is embedded in them. Not critically important to share this information with the students up front, but by the end, they should be able to see the connection.

After sharing an example or two, have students use both the Half It, Odd Ones algorithm* and then the Under, Over algorithm* to convert decimal numbers into binary. Ask them what the largest number is they can represent with the 11 positions in the table. (Answer: Add all the position values together = 2047. This can also be found by taking 10^{11} - 1 = 2047.) Ask them which algorithm they prefer and why.

Students need to know how to add numbers in binary. You can use this video (https://bit.ly/AddBinary) or you can teach this to them yourself just going over the addition rules and doing the practice problems. Be sure to double-check them in decimal form. They should also practice some examples on their own.*

Elaborate

Students should work through the proof that the Egyptian multiplication algorithm is just a combination of using binary and decimal numbers to multiply numbers. Once they have tried the example given in the algorithm, they should try another with their own starting numbers.

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Evaluate

Show students a more traditional algorithm for multiplying numbers, like the one seen in this video https://bit.ly/MultiplyBinary. Ask them to explain how this algorithm is similar and different from the Russian and Egyptian algorithms. Can they see pieces of the other algorithms in this one? Finally, they should make the case for which algorithm they will use in the future and why.

Finally, let students use the Ultimate Algorithm for Multiplication. Either have a discussion or have the write out which algorithm for multiplying they will use in the future and why.

Special Thanks

Thanks to Kat Kennewell! This lesson was inspired by a NumberPhile video featuring Johnny Ball, which Kat shared with me. She recognized the binary number connection, was sure I would love it, and she was right.

Name _____ Date _____

Russian Algorithm for Multiplying Two Numbers

An algorithm is a series of steps used to complete a task or solve a problem.

Write steps of Russian algorithm of multiplying two numbers below. You may not need to use all the space given.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____

Name _____ Date _____

Russian Algorithm Practice Problems

_____ x _____	_____ x _____
_____ x _____	_____ x _____
_____ x _____	_____ x _____

Name _____ Date _____

Egyptian Algorithm for Multiplying Two Numbers

An algorithm is a series of steps used to complete a task or solve a problem.

Write steps of Russian algorithm of multiplying two numbers below. You may not need to use all the space given.

1. _____
_____
2. _____
_____
3. _____
_____
4. _____
_____
5. _____
_____
6. _____
_____
7. _____
_____
8. _____
_____
9. _____
_____
10. _____
_____

Name _____ Date _____

Egyptian Algorithm Practice Problems

_____ x _____	_____ x _____
_____ x _____	_____ x _____
_____ x _____	_____ x _____

Name _____ Date _____

Algorithms for Converting Any Decimal Number into a Binary Number

Half It, Odd Ones (Based on the Russian algorithm for multiplying numbers.)

1. Put the decimal number you want to convert to binary in the bottom right cell.
2. Divide the number in half and place that number, without any ½'s, in the cell to the right of the first.
3. Repeat until you get to a 1.
4. Place a 0 above each even number and a 1 above each odd number.
5. You have your number in binary. Double check it by adding up the decimal place values in each place with a 1 in it.

Example for 113:

1024	512	256	128	64	32	16	8	4	2	1
				1	1	1	0	0	0	1
				1	3	7	14	28	56	113

Converting _____

1024	512	256	128	64	32	16	8	4	2	1

Converting _____

1024	512	256	128	64	32	16	8	4	2	1

Name ____ Date _____________________

Converting _______________

1024	512	256	128	64	32	16	8	4	2	1

Converting _______________

1024	512	256	128	64	32	16	8	4	2	1

Converting _______________

1024	512	256	128	64	32	16	8	4	2	1

Converting _______________

1024	512	256	128	64	32	16	8	4	2	1

Name _____ Date _____

Algorithms for Converting Any Decimal Number into a Binary Number

Under, Over (Based on the Egyptian algorithm for multiplying numbers.)

1. Move down the place values from highest to lowest until you find the two numbers between which the number you are converting resides.
2. Put a 1 in the cell just below the smaller of those two values.
3. Write the binary positional value below the 1. This will be your running total.
4. Add the next positional value to the running total.
5. If the running total is bigger than the number you are trying to convert, cross it out and put a zero above it. This does not affect your running total.
6. If the running total is smaller than the number you are converting, put a one above it.
7. Repeat steps 4-6 until you complete the conversion. If you reach the value of your number before you get to the 1s position, you can fill in the remaining positions with 0s. (And, it is always good to double check that the positional values with 1s add up to your number.)

Example for 204:

1024	512	256	128	64	32	16	8	4	2	1
			1	1	0	0	1	1	0	0
			128	128 + 64 192	~~192~~ ~~+ 32~~ ~~224~~	~~192~~ ~~+ 16~~ ~~208~~	192 + 8 200	200 + 4 204		

Converting _____

1024	512	256	128	64	32	16	8	4	2	1

Name ______________________________ Date ______________

Converting ______________

1024	512	256	128	64	32	16	8	4	2	1

Converting ______________

1024	512	256	128	64	32	16	8	4	2	1

Converting ______________

1024	512	256	128	64	32	16	8	4	2	1

Converting ______________

1024	512	256	128	64	32	16	8	4	2	1

Name _____ Date _____

Adding in Binary

To add numbers in binary, here are the rules

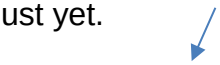
- 0 + 0 = 0
- 0 + 1 = 1
- 1 + 0 = 1
- 1 + 1 = 1 0

$\begin{array}{r} 1\ 0\ 1\ 0 \\ +\ 1\ 0\ 0\ 1 \\ \hline \end{array}$	$\begin{array}{r} 1\ 0\ 0\ 1\ 1\ 0 \\ +\ 1\ 1\ 0\ 1\ 0\ 1 \\ \hline \end{array}$
$\begin{array}{r} 1\ 0\ 1\ 1\ 0\ 1 \\ +\ 1\ 1\ 1\ 1\ 0\ 1 \\ \hline \end{array}$	$\begin{array}{r} 1\ 0\ 0\ 0\ 0\ 1 \\ 1\ 0\ 1\ 1\ 0\ 1 \\ +\ 1\ 1\ 1\ 0\ 1\ 1 \\ \hline \end{array}$

Name _____ Date _____

Proof the Egyptian Algorithm Is Just Binary Multiplication

Just follow the steps in this algorithm to multiply the two binary numbers. Do not worry about what their values are just yet.



1 1 1 0 1 x 0 1 1 0 0

1. Take the first number and put in one number per cell down the first column below the X starting with the ones place. (An arrow is pointing at it.)
2. Write the second number on the top of the grid in the five boxes, one number per box, in the order it is in.
3. Rewrite the top number in the next column down.
4. Rewrite that number in the next column down but add a 0 to the right side. So the second number, in this example, should be 0 1 1 0 0 0.
5. Rewrite the number again with an additional 0 added to the right side until each row has a number.
6. Cross out entire rows with a 0 on the left.
7. Take the binary numbers in the rows you did not cross out and add them together. Record the sum in the row just below the heavy border at the bottom.
8. Now add together the place values for each binary place with a 1 above it to determine the value of the product. You have your answer!
9. Determine the value of the two original numbers you multiplied in decimal form.
10. Multiply them together using the Egyptian algorithm from before in the space below the binary block.
11. Determine the value of the number in each row on the inside of the block and write it next to the values.
12. Can you see how the Egyptian algorithm is a combination of using binary numbers with decimal numbers to do multiplication?
13. Look back at the Russian algorithm and explain how it would be setup in binary in a slightly different way from the way the Egyptian algorithm was setup in binary using the block.

Name _____ Date _____

5-Digit Multiplication Block for Binary Multiplication

X										
1024	512	256	128	64	32	16	8	4	2	1

Egyptian Algorithm

_____ X _____

Name _____ Date _____

5-Digit Multiplication Block for Binary Multiplication

X										
1024	512	256	128	64	32	16	8	4	2	1

Egyptian Algorithm

_____ X _____

Name _____ Date _____

Ultimate Multiplication Algorithm

Combining decimal and binary algorithms for a faster method.

1. Place first number in the top cell of the middle column.
2. Divide that number in half and write the new number in the next cell down, dropping any ½ remainders.
3. Continue until you reach 1.
4. In the second column, write a 1 next to any odd numbers and a 0 next to any even numbers.
5. Put the second number at the top of the third column.
6. Double it in each new cell as you move down the column until you reach the last 1.
7. Cross out entire rows with 0's in the first column.
8. Add up the numbers remaining on the right, and you have your answer!

Example: 145 x 12 = ??

1	1	145	12
~~2~~	~~0~~	~~72~~	~~24~~
~~4~~	~~0~~	~~36~~	~~48~~
~~8~~	~~0~~	~~18~~	~~96~~
16	1	9	192
~~32~~	~~0~~	~~4~~	~~384~~
~~64~~	~~0~~	~~2~~	~~786~~
128	1	1	1536
256			
512			
1024			

$$\begin{array}{r} 12 \\ 192 \\ +1536 \\ \hline 1740 \end{array}$$

Name _____ Date _____

Decimal/Binary Multiplication Blocks

_____ X _____			
1			
2			
4			
8			
16			
32			
64			
128			
256			
512			
1024			

_____ X _____			
1			
2			
4			
8			
16			
32			
64			
128			
256			
512			
1024			

Binary #MicDropMath Multiply This - Answer Key

Russian Algorithm for Multiplying Two Numbers

1. Write the two numbers you want to multiply side-by-side with an “x” in between them.
2. Divide the number on the left in half and write that number below it. Ignore any ½ remainders.
3. Repeat Step 2 until you get to the number 1.
4. Double the number on the right and write that number below it until there is a number in each row.
5. Cross out entire rows with even numbers in the left column.
6. Add up the numbers in the right column which have not been crossed out and you have your answer!

Binary #MicDropMath Multiply This - Answer Key

Egyptian Algorithm for Multiplying Two Numbers

1. Write the two numbers you want to multiply side-by-side with an “x” in between them.
2. Write the number on the right exact as it is below itself.
3. Write a 1 below the number on the left.
4. Double the 1 and record the new number below the one.
5. Keep doubling until you get to a number that is bigger than the number at the top of the left column. Erase or cross out that number.
6. Double the number on the right and write that number below it until there is a number in each row.
7. Determine which numbers in the left column add up to the number at the top of the column.
8. Cross out the entire rows with any numbers on the left which are not used to total the number on the top.
9. Add up the numbers in the right column which have not been crossed out and you have your answer!