

Binary Math “Magic” Lesson

Materials:

Advanced Version of the Binary Numbers Math “Magic” Trick (0-31 version)
Advanced Version of the Binary Numbers Math “Magic” Trick (0-63 version)
(available here: <https://stemazing.org/binary-number-trick/>)
Graph paper for taking notes (you might consider having them make STEMazing Journals like these: <https://stemazing.org/stemazing-journal-hack/>)
Pens/Pencils

You will need one set of the 0-31 cards per student. Sort them into stacks of cards that are the same – all the cards with 1 in the top left corner together, all the cards with 2 in the top left corner together, and so on.

You will need one set of the 0-63 cards per student. Keep these in complete sets of six cards.

It is best to print the cards on two different colors of paper so they can be easily sorted.

Engage:

Write the number 10011011 on the board. Ask your students to write down in their journals the value the number represents. They can compare their answer with others and should come up with – ten million eleven thousand eleven. After everyone has agreed on the value, explain that it actually has a value of 155 and they will know why by the end of class today.

Explore:

Explain to students: “Noticing and describing patterns in math and science is a critical skill. Today we are going to be looking for patterns on the cards I am giving you. I want you to write any patterns you notice in your journal and then I will give you some time to share what you notice with others at your table.”

Hand every student JUST the first card from the sets. (Pictured at right.) First, give them about 3-5 minutes to write silently in their journal. You may prompt them with questions like: What patterns do you notice? What numbers are on the card? What numbers are not on the card? After they have made their own individual observations, let them share out what they noticed in small groups of 3-5 students. Once students have discussed the patterns in their small groups, facilitate a share-out by allowing

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

each group to share something they noticed until you have exhausted patterns and observations made by the class.

Now, hand every student the SECOND card from the sets. (Pictured at right.) Let them keep the first one as well. Again, repeat the 3-5 minutes of silent observation of patterns, which they should write in their journals, followed by sharing with their small group. You may prompt them with questions like: What patterns do you notice on the second card? What number are on the card? What numbers are not on the card? What numbers does it have in common with the first card? Once students have discussed the patterns in their small groups, facilitate a share-out by allowing each group to share something they noticed until you have exhausted patterns and observations made by the class.

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

Now have students make a prediction. What do they think the numbers on the third card are going to be? Let them discuss with their small groups and then share out their ideas with the class. Do not hand out the third card until they have made a prediction. It is most likely true that they will predict it will start with a 3 unless they have seen this trick before.

Now, hand every student the THIRD card from the sets. (Pictured at right.) Allow them to discuss patterns in their small groups but be sure they also record what they notice in their journals. Many will be surprised the card did not have a 3 in the top left square. After they have discussed patterns on this card, let them again share out with the entire class. Before handing them the next card, see if they can predict all or some of the numbers they expect to see on the fourth card. They should draw their prediction in their journals.

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

Hand every student the FOURTH card from the sets. (Pictured at right.) Give them some time to discuss whether their predictions were partially or completely correct. Have them note any patterns they see on this card. Explain that there is only one card left in the set. Ask them to, again, predict what numbers the last card will have on it and record their prediction in their journal.

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

Finally, hand out the last card in the set. (Pictured at right.) Give them some time to discuss whether their prediction matched the actual card. Now that they have all five cards, ask them if they can find any numbers which appear on just one and only one card. What do they notice about these numbers? (It is possible someone may have noticed this before this point as well.) They should be able to figure out that 1, 2, 4, 8, and 16 only appear on one card.

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

It is time to wow them with the binary math “magic” trick. Take a complete set of cards or use the PowerPoint Version 1-31 presentation found here <https://stemazing.org/binary-number-trick/>. Pick one student to think about, but not share aloud, the number for the day they were born. Tell them to ignore the month and year and just think about the number for the day they were born. Now, ask them to simply view each card and answer “yes” or “no” indicating if they see their number on the card or not. Keep a running total of the top left hand numbers on the cards to which they say “yes”. This value is the number for the day they were born. See examples below. You should do the trick on two or three students to prove it was not a fluke.

If they were to say “yes” to the following cards and “no” to the other two:

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

Then, they were born on the 13th of the month – $8 + 4 + 1 = 13$
This is the sum of the numbers in the top left corner of the cards to which they said yes, indicating their number was on that card.

Another example, if they said “yes” their number is on the following cards and “no” to the others:

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

Then, they were born on the 20th of the month – $16 + 4 = 20$

Should they say “yes” to only one card, then their birth day is simply the number in the top left corner of that card.

At this point, you can decide if you are going to immediately share the trick with them or make them figure it out themselves. You could ask them to think about why you would want to know if their number is on the card. It must be that the cards to which you say “yes” allow you to, somehow, figure out the number. You could select a few more students and have the class move the yes cards together. Tell them what the number is and see if they can figure out how you get it. As students figure it out, ask them not to share the secret with the class but rather, let you think of a number and test if they really do get it by answering “yes” or “no” to their cards. This could either be done as whole class or small groups.

After a sufficient amount of productive struggle, you should let the entire class in on the secret and let them practice on each other – one student thinking of a number between 1-31 and the other attempting to figure it out.

Explain:

Tell students this math “magic” trick is based on the binary number system. To explain the binary system, show this video until 3:45 when it becomes an advertisement for the sponsor: <http://bit.ly/BinaryNumbersVideo>

The video will cover positional notation, binary numbers, base ten, numeral base systems, alphanumeric characters, and URL shorteners. List these vocabulary words on the board and have students watch the video once all the way through listening for these terms in particular. Then, watch the video again pausing after each word is referenced, and try to come up with a definition for each term. This could be discussed and decided upon in their small groups and then shared out with the class.

Binary Numbers – a numeral base system which uses only two numbers – 1 and 0

Base Ten – a numerals system which uses ten symbols, the digits 0-9

Numeral Base Systems – systems which uses only the digits 0-9 or a subset of those digits to represent different values – examples are base 10 (which we use every day), base 2 or binary which uses just two values, but there are also base three, base four, and so on.

Positional Notation – addition of new locations to the left of the initial position once when its maximum value has been reached. Each new position has a value which is a factor of the number of symbols you are using greater than the position to its right.

Alphanumeric Characters – systems which use both numbers and letters

URL Shorteners – take long website URLs and shorten them using alphanumeric strings to give them a unique, but short, URL



Elaborate:

Now, give each student a complete set of the Advanced Version of the Binary Numbers Math “Magic” Trick (0-63 version). They should be given time to discuss how these cards are similar and different from the first set of cards they received. You can ask them: Do the patterns they noted in the first set of cards still hold?

Have students practice using these on each other with one student thinking of a number between 1-63 and the other student figuring it out. This works the same way as the first set of cards – just simply add up the top left corner numbers on the cards they say “yes” to and the sum is the number they picked.

Any of the lessons from CSunplugged in the Binary numbers unit would be a great extension to this lesson. Find those lessons with printables, directions, and videos for each lesson here: <https://csunplugged.org/en/topics/binary-numbers/unit-plan/>

Do not be fooled by the low age range. Most adults do not know this stuff. This unit includes lesson on the following:

- How binary digits work
- Reinforcing sequencing in binary number systems
- Codes for letters using binary representation

Evaluation:

Students can be challenged to take the binary math “magic” trick home with them to show off their number devising skills with their families.

You could give them binary numbers to evaluate and see if they can actually come up with the correct value.

As students still do not generally understand where the numbers on the specific cards actually come from, it is recommended that you use the Binary Math “Magic” Build Your Own Trick Cards lesson to let them discover how you determine which numbers go on each card used for this trick. This lesson allows students to just replicate the 0-31 cards or they can challenge themselves with developing the 0-63 or even 0-127 cards.

If they can then explain how this processed work and/or give you the binary number for any value in the range they are exploring, you can rest assured they have learned how the binary system actually works. It is important to note to students that binary is really just a “code” used to send messages from one place to another. This would be a great time to introduce them to the ASCII binary codes. (See CSunplugged unit linked above for lesson ideas.)